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Mass of the Fayet hypermultiplet induced by a central charge constraint

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ABSTRACT: We show that the mass of the Fayet hypermultiplet, which represents the matter sector of $N = 2$ supersymmetric Yang-Mills theory, may be induced through a generalization of the central charge constraint usually proposed in the literature. This mass showing up as a parameter of the supersymmetry transformations, we conclude that it will stay unrenormalized at the quantum level.

KEYWORDS: Renormalization Regularization and Renormalons, Extended Supersymmetry, Superspaces.

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Contents

1. Introduction

In $N = 2$ supersymmetric Yang-Mills theories, matter, i.e. spin 0 and 1/2 particles, is represented by the Fayet "hypermultiplet" [1, 2]. It is represented in superspace by a certain constrained superfield. One of these constraints, necessary in order to render finite the number of local field components of the superfield, concerns the central charge [3, 2, 4, 5]. We propose in the present paper a generalization of this constraint, involving a complex parameter λ of mass dimension 1, which will appear in the supersymmetry transformation rules of the component fields and show up in the resulting invariant action as a contribution to the physical mass. An interesting consequence of this construction is a nonrenormalization theorem for the mass, in models where the latter is entirely due to the constraint parameter λ, since the latter may be defined as a parameter of the supersymmetry transformations laws.

It is worthwhile recalling that the masses of the matter particles in $N = 2$ supersymmetric Yang-Mills theories may be generated by coupling the matter fields to a constant abelian super-Yang-Mills field strength, the values of the masses being proportional to the value of this field strength [6, 7]. Thus, the mass generation via a generalized central charge constraint proposed in the present paper offers an alternative way of generating the masses, with the parameter λ replacing the constant field strength. The interest of this alternative way is that it appears more natural, being purely algebraic. In our scheme, indeed, λ parametrizes the generalized central charge constraint, which is algebraic.

JHEP03(2002)006

In this paper, we shall restrict ourselves to the construction of the free theory of one hypermultiplet in order to explain the mechanism in a simple way. The construction of the full $N = 2$ theory with coupling to a gauge supermultiplet is left for a forthcoming publication [8].

The plan is the following. After recalling, in section 2, some notations and definitions for $N = 2$ super-Yang-Mills theory in the superspace formalism, we give the construction of the Fayet matter supermultiplet in section 3, using the generalized supercharge constraint. We then construct the action in section 4, showing the generation of the mass. The discussion of the nonrenormalization of this mass is performed in section 5. Our conclusions are presented in section 6.

2. $N = 2$ superspace

$N = 2$ supersymmetry is defined by the Wess-Zumino superalgebra [2, 9]

$$[\mathcal{P}_A, \mathcal{P}_B] = T^C_{AB}\mathcal{P}_C\,, \tag{2.1}$$

where $\mathcal{P}_A = \{P_a,\, Q^i_\alpha,\, \bar{Q}_{i\dot\alpha},\, Z,\, \bar{Z}\}$ is the set of infinitesimal generators: the translations P_a, $(a = 0, \ldots, 3)$, the supersymmetries Q^i_α, $\bar{Q}_{i\dot\alpha}$ — the Lorentz spin indices α and $\dot\alpha$ taking the values $1, 2$, as well as the isospin SU(2) indices i — and the complex central charge Z, $\bar{Z}$. Notations and conventions are explained in the Appendix. Under the Lorentz transformations, P transforms as a vector, Q and $\bar{Q}$ as Weyl spinors — in the Lorentz group representations $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$, characterized by the indices α and $\dot\alpha$, respectively. Z and $\bar{Z}$ are scalars. Moreover Q and $\bar{Q}$ also transform as doublets of the isospin group SU(2) — acting on the index i.

The generators P, Z and $\bar{Z}$ are bosonic, whereas Q and $\bar{Q}$ are fermionic. Accordingly, the bracket $[\cdot, \cdot]$ in the l.h.s. of (2.1) is a graded commutator, i.e. an anticommutator if both entries are fermionic, and a commutator otherwise.

Finally, the structure constants of the superalgebra (2.1) — the "torsions" — are given by:

$$T^{i\,j\,z}_{\alpha\beta} = 2i\varepsilon^{ij}\varepsilon_{\alpha\beta}\,, \qquad T^{i\,j\,\bar{z}}_{\dot\alpha\dot\beta} = 2i\varepsilon^{ij}\varepsilon_{\dot\alpha\dot\beta}\,, \qquad T^{i\,j\,a}_{\alpha\dot\beta} = -2i\varepsilon^{ij}\sigma^a_{\alpha\dot\beta}\,, \tag{2.2}$$

all the other torsion coefficients vanishing.

2.1 Representation of the $N = 2$ Wess-Zumino algebra

Our first task is to define the superspace representation of the Wess-Zumino algebra (2.1). $N = 2$ superspace [2, 9] is described by the coordinates $\{X^A\} = \{x^a, \theta^i_\alpha, \bar\theta_{i\dot\alpha}, z, \bar{z}\}$. These coordinates are, respectively, the space-time coordinates, Weyl isospin coordinates and its conjugate, complex central charge and its conjugate.[1] The spinor coordinates, θ and $\bar\theta$, are Grassmann (i.e. anticommuting or "fermionic") numbers, the remaining ones are

[1] See the appendix for our conventions of complex conjugation. As for the central charge coordinate, we have $\bar{z} = -z^*$, where z^* is the complex conjugate of z.

ordinary (i.e. commuting or "bosonic") numbers, so the manifold coordinates satisfy the (anti)commutation rules:

$$[X^A, X^B] \equiv X^A X^B - (-)^{ab} X^B X^A = 0\,, \tag{2.3}$$

where the grading $a = 0$ if X^A is bosonic, and $a = 1$ in the fermionic case.

A superfield is a function in superspace, $\phi(X)$, transforming under the generators of the superalgebra (2.1) as follows:

$$\begin{aligned}
P_a\phi &= \frac{\partial}{\partial x^a}\phi\,, \\
Q^i_\alpha\phi &= \left(\frac{\partial}{\partial\theta^a_i} - i\partial_{\alpha\dot\beta}\bar\theta^{i\dot\beta} + \theta^i_\alpha\frac{\partial}{\partial z}\right)\phi\,, \\
\bar Q_{i\dot\alpha}\phi &= \left(-\frac{\partial}{\partial\bar\theta^{i\dot\alpha}} + i\theta^\alpha_i\partial_{\alpha\dot\alpha} - \bar\theta_{i\dot\alpha}\frac{\partial}{\partial\bar z}\right)\phi\,, \\
Z\phi &= \frac{\partial}{\partial z}\phi\,, \qquad \bar Z\phi = \frac{\partial}{\partial\bar z}\phi\,.
\end{aligned} \tag{2.4}$$

where we have defined

$$\partial_{\alpha\dot\alpha} \equiv \sigma^a_{\alpha\dot\alpha}\partial_a\,, \qquad \bar\partial_{\dot\alpha\alpha} \equiv \bar\sigma^a_{\dot\alpha\alpha}\partial_a\,. \tag{2.5}$$

This provides the superfield representation of the superalgebra (2.1).

The covariant derivatives D_A are superspace derivatives defined such that $D_A\phi$ transform in the same way as the superfield ϕ itself. They are given by

$$\begin{aligned}
D_a\phi &= \frac{\partial}{\partial x^a}\phi\,, \\
D^i_\alpha\phi &= \left(\frac{\partial}{\partial\theta^a_i} + i\partial_{\alpha\dot\beta}\bar\theta^{i\dot\beta} - \theta^i_\alpha\frac{\partial}{\partial z}\right)\phi\,, \\
\bar D_{i\dot\alpha}\phi &= \left(-\frac{\partial}{\partial\bar\theta^{i\dot\alpha}} - i\theta^\alpha_i\partial_{\alpha\dot\alpha} + \bar\theta_{i\dot\alpha}\frac{\partial}{\partial\bar z}\right)\phi\,, \\
D_z\phi &= \frac{\partial}{\partial z}\phi\,, \qquad D_{\bar z}\phi = \frac{\partial}{\partial\bar z}\phi\,.
\end{aligned} \tag{2.6}$$

and obey the same (anti)commutation rules as the generators, up to the signs of the right-hand sides:

$$[D_A, D_B] = -T^C_{AB}D_C\,, \tag{2.7}$$

the torsion coefficients T being given in (2.2).

The components of the supermultiplet corresponding to the superfield ϕ are the coefficients of its expansion in powers of θ and $\bar\theta$. A generic component can be written as

$$C_n = (D)^n\phi|\,, \tag{2.8}$$

where $(D)^n$ is some product of D^i_α and $\bar D_{i\dot\alpha}$ derivatives, and where the symbol $|$ means that the expression is evaluated at $\theta = \bar\theta = 0$. It follows from this remark and from the explicit transformation rules (2.4), that the action of the supersymmetry and central charge generators on the components can be written as [3]

$$\begin{aligned}
Q^i_a C_n &= D^i_\alpha(D)^n\phi|\,, \qquad & \bar Q_{i\dot\alpha}C_n &= D_{i\dot\alpha}(D)^n\phi|\,, \\
ZC_n &= \partial_z(D)^n\phi|\,, \qquad & \bar ZC_n &= \partial_{\bar z}(D)^n\phi|\,.
\end{aligned} \tag{2.9}$$

3. Construction of the free Fayet hypermultiplet

The Fayet hypermultiplet

$$\phi_i \equiv (\phi_i, \chi_\alpha, \bar{\psi}_{\dot{\alpha}}, F_i) \tag{3.1}$$

is formed by two SU(2) doublets of complex scalars (ϕ_i, F_i) and two Weyl spinors $(\bar{\psi}_{\dot{\alpha}}, \chi_a)$. It represents the matter sector of $N = 2$ supersymmetric Yang-Mills theories [2], but we shall only consider the free Fayet hypermultiplet in the present paper. It may be represented by an SU(2) doublet of complex superfield[2] $\phi_i(X)$ subjected to the supersymmetric constraints

$$D^i_\alpha \phi^j + D^j_\alpha \phi^i = 0\,, \qquad \bar{D}_{i\dot{\alpha}} \phi^j + \bar{D}^j_{\dot{\alpha}} \phi_i = 0\,. \tag{3.2}$$

3.1 Central charge constraint and supersymmetry transformations

The dependence of the superfield on the central charge coordinate z leads in general to an infinity of local field components. In order to define a finite supersymmetry representation, one has to impose a central charge constraint which restricts the dependence on z and $\bar{z}$. We shall choose the constraint to be

$$(\partial_z - e^{iw}\partial_{\bar{z}})\phi_i = \lambda\phi_i\,, \qquad (\partial_{\bar{z}} - e^{-iw^*}\partial_z)\bar{\phi}^i = -\lambda^*\bar{\phi}^i\,. \tag{3.3}$$

It depends on a complex parameter λ of the dimension of a mass and on a dimensionless complex "phase" parameter w. This constraint generalizes the one found in the literature [4], which corresponds to zero λ and w. We remark that, since D^i_α and $\bar{D}^i_{\dot{\alpha}}$ commute with ∂_z and $\partial_{\bar{z}}$, the constraint above holds for the superfield $\phi_i(\bar{\phi}^i)$ and all its derivatives, in particular on the derivatives which define the component fields (2.8).

In order to establish the supersymmetry transformation rules of the hypermultiplet components, we first define the latters by the following covariant derivatives of the superfield ϕ:

$$\phi_i \equiv \phi_i|\,, \qquad \chi_a \equiv \frac{1}{2\sqrt{2}} D^i_\alpha \phi_i\Big|\,, \qquad \bar{\psi}_{\dot{\alpha}} \equiv \frac{1}{2\sqrt{2}} \bar{D}^i_{\dot{\alpha}} \phi_i\Big|\,, \qquad F^j \equiv \frac{i}{8} D^{i\alpha} D_{i\alpha} \phi^j\Big| = \partial_z \phi^j\,|\,. \tag{3.4}$$

The last equality in the definition of F^i follows from the Fayet constraints (3.2) and the (anti)commutation rules (2.7).

Using formula (2.9) together with the (anti)commutation rules (2.7), the Fayet constraints (3.2) and the generalized constraint (3.3), we can find (λ, w)-dependent transformation rules for the components, obeying the superalgebra (2.1). However, before doing that, let us observe that the *real part* of the "phase" w may be eliminated through certain redefinitions. Indeed, writing

$$w = u + iv\,, \qquad \text{with } u, v \text{ real}\,, \tag{3.5}$$

we first note that the constraint (3.3) reduces to the form

$$(\partial_z - e^{-v}\partial_{\bar{z}})\phi_i = \lambda\phi_i\,, \qquad (\partial_{\bar{z}} - e^{-v}\partial_z)\bar{\phi}^i = -\lambda^*\bar{\phi}^i\,, \tag{3.6}$$

[2]The same symbol ϕ represents the multiplet (3.1), the corresponding superfield, as well as the first component of the latter, i.e. its value at $\theta = \bar{\theta} = 0$.

if one redefines the central charge coordinates and the parameter λ according to

$$z \to e^{iu/2} z\,, \qquad \bar{z} \to e^{-iu/2} z\,, \qquad \lambda \to e^{-iu/2}\,. \tag{3.7}$$

However, compatibility of this redefinition with the covariant derivative superalgebra of (2.7) implies that the remaining superspace coordinates has to be redefined, too:

$$\theta \to e^{iu/4}\theta\,, \qquad \bar{\theta} \to e^{-iu/4}\bar{\theta}\,, \qquad x \to x\,. \tag{3.8}$$

This amounts to a redefinition of the supercovariant derivatives as

$$D^i_\alpha \to e^{-iu/4} D^i_\alpha\,, \qquad \bar{D}_{i\dot{\alpha}} \to e^{iu/4}\bar{D}_{i\dot{\alpha}}\,, \tag{3.9}$$

or, in view of (3.4), to phase redefinitions of the component fields:

$$\begin{array}{llll} \phi \to \phi\,, & \chi \to e^{-iu/4}\chi\,, & \bar{\psi} \to e^{iu/4}\bar{\psi}\,, & F \to e^{-iu/2}F\,, \\ \bar{\phi} \to \bar{\phi}\,, & \bar{\chi} \to e^{iu/4}\bar{\chi}\,, & \psi \to e^{-iu/4}\psi\,, & \bar{F} \to e^{iu/2}\bar{F}\,. \end{array} \tag{3.10}$$

Using now formula (2.9) together with the Fayet constraints (3.2) and the generalized central charge constraint in the form (3.6), we find the following (λ, v)-dependent transformation rules for the components:

$$\begin{aligned} Q^i_\alpha \phi_j &= \sqrt{2}\delta^i_j \chi_\alpha\,, & \bar{Q}_{i\dot{\alpha}}\phi^j &= -\sqrt{2}\delta^j_i \bar{\psi}_{\dot{\alpha}}\,, \\ Q^i_\alpha \chi_\beta &= -\sqrt{2}i\varepsilon_{\alpha\beta}F^i\,, & \bar{Q}_{i\dot{\alpha}}\chi_\alpha &= -\sqrt{2}i\partial_{\alpha\dot{\alpha}}\phi_i\,, \\ Q^i_\alpha \bar{\psi}_{\dot{\beta}} &= \sqrt{2}i\partial_{\alpha\dot{\beta}}\phi^i\,, & \bar{Q}_{i\dot{\alpha}}\bar{\psi}_{\dot{\beta}} &= e^v\sqrt{2}i\varepsilon_{\dot{\alpha}\dot{\beta}}(F_i - \lambda\phi_i)\,, \\ Q^i_\alpha F_j &= \sqrt{2}\delta^i_j\left(e^{-v}\partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + \lambda\chi_\alpha\right), & \bar{Q}_{i\dot{\alpha}}F^j &= -\sqrt{2}\delta^j_i\partial_{\alpha\dot{\alpha}}\chi^\alpha\,, \end{aligned} \tag{3.11}$$

In the same way one finds the transformations for the conjugated hypermultiplet $\bar{\phi}^i = (\bar{\phi}^i, \bar{\chi}_{\dot{\alpha}}, \psi_\alpha, \bar{F}^i)$:

$$\begin{aligned} Q^i_\alpha \bar{\phi}_j &= \sqrt{2}\delta^i_j\psi_\alpha & \bar{Q}_{i\dot{\alpha}}\bar{\phi}^j &= \sqrt{2}\delta^j_i\bar{\chi}_{\dot{\alpha}} \\ Q^i_\alpha \bar{\chi}_{\dot{\beta}} &= -\sqrt{2}i\partial_{\alpha\dot{\beta}}\bar{\phi}^i & \bar{Q}_{i\dot{\alpha}}\bar{\chi}_{\dot{\beta}} &= \sqrt{2}i\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{F}_i \\ Q^i_\alpha \psi_\beta &= e^v\sqrt{2}i\varepsilon_{\alpha\beta}(\bar{F}^i - \lambda^*\bar{\phi}^i) & \bar{Q}_{i\dot{\alpha}}\psi_\alpha &= -\sqrt{2}i\partial_{\alpha\dot{\alpha}}\bar{\phi}^i \\ Q^i_\alpha \bar{F}_j &= \sqrt{2}\delta^i_j\partial_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} & \bar{Q}_{i\dot{\alpha}}\bar{F}^j &= \sqrt{2}\delta^j_i(e^{-v}\bar{\partial}_{\dot{\alpha}\alpha}\psi^\alpha + \lambda^*\bar{\chi}_{\dot{\alpha}}) \end{aligned} \tag{3.12}$$

Finally, the central charge tranformation laws are

$$\begin{aligned} Z\phi^i &= F^i\,, & \bar{Z}\phi^i &= e^v(F^i - \lambda\phi^i)\,, \\ Z\chi_\alpha &= e^{-v}\partial_{\alpha\dot{\beta}}\bar{\psi}^{\dot{\beta}} + \lambda\chi_\alpha\,, & \bar{Z}\chi_\alpha &= \partial_{\alpha\dot{\beta}}\bar{\psi}^{\dot{\beta}}\,, \\ Z\bar{\psi}_{\dot{\beta}} &= \partial_{\alpha\dot{\beta}}\chi^\alpha\,, & \bar{Z}\bar{\psi}_{\dot{\beta}} &= e^v(\partial_{\alpha\dot{\beta}}\chi^\alpha - \lambda\bar{\psi}_{\dot{\beta}})\,, \\ ZF^i &= e^{-v}\Box\phi^i + \lambda F^i\,, & \bar{Z}F^i &= \Box\phi^i\,, \end{aligned} \tag{3.13}$$

JHEP03(2002)006

and similarly for the conjugate multiplet $\bar\phi^i$:

$$
\begin{aligned}
\bar Z\bar\phi^i &= -\bar F^i\,, & Z\bar\phi^i &= -e^v(\bar F^i - \lambda^*\bar\phi^i)\,,\\
\bar Z\bar\chi_{\dot\alpha} &= -e^{-v}\partial_{\dot\alpha\beta}\psi^\beta - \lambda^*\bar\chi_{\dot\alpha}\,, & Z\bar\chi_{\dot\alpha} &= -\partial_{\dot\alpha\beta}\psi^\beta\,,\\
\bar Z\psi_\alpha &= -\partial_{\alpha\dot\beta}\bar\chi^{\dot\beta}\,, & Z\psi_\alpha &= -e^v(\partial_{\alpha\dot\beta}\bar\chi^{\dot\beta} - \lambda^*\psi_\alpha)\,,\\
\bar Z\bar F^i &= -e^{-v}\Box\bar\phi^i - \lambda^*\bar F^i\,, & Z\bar F^i &= -\Box\bar\phi^i\,.
\end{aligned}
\tag{3.14}
$$

One readily checks that the equations (3.13) and (3.14) imply that for the superfield ϕ ($\bar\phi$) or any of its components C ($\bar C$), the central charge constraints (3.6) are satisfied, namely:

$$
(\partial_z - e^{-v}\partial_{\bar z})C = \lambda C\,, \qquad (\partial_{\bar z} - e^{-v}\partial_z)\bar C = -\lambda^*\bar C\,, \tag{3.15}
$$

as it should be by construction.

Despite of the (v,λ)-dependence of the transformations (3.11)–(3.14), it is simple to verify that the superalgebra algebra closes accordingly to (2.1), independently of these parameters. So, v and λ are completely free parameters, and λ may be complex.

4. The hypermultiplet lagrangian

In order to get the lagrangian of the hypermultiplet constructed in the preceding section, we may use an algorithm due to Hasler, based on the

Proposition. *[5]. Let be a superfield polynomial L^{ij} — called the "kernel" — satisfying the conditions of zero symmetric derivatives*

$$
D_\alpha^{(i}L^{jk)} = 0\,, \qquad \bar D_{\dot\alpha}^{(i}L^{jk)} = 0\,. \tag{4.1}
$$

Then the superfields

$$
L \equiv -D_k^\alpha\Lambda_\alpha^k\,, \qquad \bar L \equiv -D_{k\dot\alpha}\bar\Lambda^{k\dot\alpha}\,, \tag{4.2}
$$

where

$$
\Lambda_\alpha^k \equiv D_{i\alpha}L^{ik}\,, \qquad \bar\Lambda^{k\dot\alpha} \equiv \bar D_i^{\dot\alpha}L^{ik}\,, \tag{4.3}
$$

transform under supersymmetry — with infinitesimal parameters ξ, $\bar\xi$ — as

$$
\begin{aligned}
\delta L &= i\partial_z\left(\xi_i^\alpha\Lambda_\alpha^i + \bar\xi_{i\dot\alpha}\bar\Lambda^{i\dot\alpha}\right) - 2i\partial_a\left(\bar\xi_{i\dot\alpha}\bar\sigma^{a\,\dot\alpha\beta}\Lambda_\beta^i\right)\,,\\
\delta\bar L &= -i\partial_{\bar z}\left(\xi_i^\alpha\Lambda_\alpha^i + \bar\xi_{i\dot\alpha}\bar\Lambda^{i\dot\alpha}\right) - 2i\partial_a\left(\xi_i^\alpha\sigma^a_{\alpha\dot\beta}\bar\Lambda^{idb}\right)\,.
\end{aligned}
\tag{4.4}
$$

Let us apply this proposition to the kernel

$$
L^{ij} = ie^{i\gamma}\partial_{\bar z}\bar\phi^j\phi^i + i\bar\phi^i\partial_z\phi^j\,, \tag{4.5}
$$

where γ is an arbitrary complex "phase".[3] The conditions (4.1) are satisfied due to the Fayet constraints (3.2) obeyed by both ϕ^i and $\bar\phi^i$. We shall consider a linear combination of the superfield defined by (4.2) and of its complex conjugate:

$$
L_\theta \equiv L + e^{i\theta}\bar L\,, \tag{4.6}
$$

[3]This phase slightly generalizes the kernel proposed in the literature [2, 4].

where θ is a complex number. Since ξ and $\bar{\xi}$ are independent parameters, it clearly follows from (4.4) that necessary and sufficient conditions for the supersymmetric variation of L_θ to be a total divergence are[4]

$$(\partial_z - e^{i\theta}\partial_{\bar{z}})\Lambda^i_\alpha = \text{total divergence}\,, \qquad (\partial_z - e^{i\theta}\partial_{\bar{z}})\bar{\Lambda}^i_{\dot{\beta}} = \text{total divergence}\,. \tag{4.7}$$

The kernel L^{ij}, and consequently Λ and $\bar{\Lambda}$, being bilinear in ϕ^i and $\bar{\phi}^i$, we easily check, using the equations (3.15), (4.3), (3.13), (3.14) and (3.6), that the condition (4.7) is satisfied if

$$v = o\,, \qquad \theta = \gamma = 0\,, \qquad \lambda^* = -\lambda\,. \tag{4.8}$$

We finally get a supersymmetric lagrangian — invariant up to a total derivative — as:

$$\mathcal{L} = \frac{i}{24}L_\theta + \text{c.c.} = \mathcal{L}_{\text{cin}} + \mathcal{L}_\lambda\,, \tag{4.9}$$

with

$$\begin{aligned} \mathcal{L}_{\text{cin}} &= \bar{F}F - \partial_a\bar{\phi}\partial^a\phi - i\chi\partial\!\!\!/\bar{\chi} - i\psi\partial\!\!\!/\bar{\psi}\,, \\ \mathcal{L}_\lambda &= -\frac{\lambda}{2}\left(\bar{F}\phi + i\bar{\chi}\bar{\psi} + i\psi\chi - \bar{\phi}F\right)\,, \end{aligned} \tag{4.10}$$

where we have used the notation $(\partial\!\!\!/\bar{\psi})_\alpha = \partial_{\alpha\dot{\beta}}\bar{\psi}^{\dot{\beta}} = \sigma^a_{\alpha\dot{\beta}}\partial_a\bar{\psi}^{\dot{\beta}}$.

Let us note that the central charge constraint reads now

$$(\partial_z - \partial_{\bar{z}})\phi_i = \lambda\phi_i\,, \qquad (\partial_{\bar{z}} - \partial_z)\bar{\phi}^i = -\lambda^*\bar{\phi}^i\,, \qquad \text{with } \lambda^* = -\lambda\,. \tag{4.11}$$

Although the transformation rules (3.11)–(3.14) constitute a representation of the superalgebra (2.1) for any real v and complex λ, only the values $v = 0$ and λ imaginary allow for an invariant lagrangian.

We observe that the terms in λ are mass terms, which have been induced from the supersymmetry transformation rules we have defined in the last Section. Of course, it is still possible to add a mass term "by hand". This can be done with Hasler's algoritm, too. We find in this way the invariant mass lagrangian

$$\mathcal{L}_\mu = \mu\left(\bar{F}\phi + \bar{\phi}F + i\bar{\chi}\bar{\psi} - i\psi\chi\right)\,, \tag{4.12}$$

where μ is a real mass parameter.

4.1 Equations of motions and masses

The total lagrangian reads

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{cin}} + \mathcal{L}_{\text{mass}}\,, \tag{4.13}$$

where

$$\mathcal{L}_{\text{mass}} = M\left(\bar{F}\phi + i\bar{\chi}\bar{\psi}\right) + M^*\left(\bar{\phi}F - i\psi\chi\right)\,, \qquad \text{with } M = \mu - \frac{1}{2}\lambda\,, \quad M^* = \mu + \frac{1}{2}\lambda\,, \tag{4.14}$$

[4]Ref. [5] considers the less general situation where $e^{i\theta} = \pm 1$ and $(\partial_z - e^{i\theta}\partial_{\bar{z}})(\xi^\alpha_i\Lambda^i_\alpha + \bar{\xi}_{i\dot{\alpha}}\bar{\Lambda}^{i\dot{\alpha}}) = 0$.

implying the equations of motions

$$\begin{aligned} F_i + M\phi_i &= 0\,, & \partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} + M^*\chi_\alpha &= 0\,,\\ \Box\phi_i + M^* F_i &= 0\,, & \bar\partial^{\dot\alpha\alpha}\chi_\alpha - M\bar\psi^{\dot\alpha} &= 0\,. \end{aligned} \tag{4.15}$$

These equations generalize the ones found in the literature by the presence of the parameter λ, which contributes to the mass M. Despite of μ and λ being complex, there is no tachyon in the theory. In fact, it is simple to verify that all component fields satisfy Klein-Gordon equations with a real mass $|M|$:

$$\left(-\Box + |M|^2\right)\varphi = 0\,, \qquad \varphi = \phi,\, F, \psi,\, \chi\,, \qquad |M|^2 = \mu^2 + \frac{1}{4}|\lambda|^2\,. \tag{4.16}$$

4.2 Parity

The mass lagrangian (4.14) in general is not invariant under parity invariance, defined by the transformations[5]

$$\begin{aligned} (x^0, \mathbf{x}) &\to (x^0, -\mathbf{x})\,,\\ (\phi^i, \chi^\alpha, \bar\psi_{\dot\beta}, F^i) &\leftrightarrow (\bar\phi_i, \bar\chi_{\dot\alpha}, \psi^\beta, -\bar F_i)\,. \end{aligned} \tag{4.17}$$

This invariance however holds if (and only if) M is purely imaginary, i.e.:

$$\mu = 0\,. \tag{4.18}$$

5. Nonrenormalization of the mass

The complex mass coefficient M of the general mass term (4.14) has two contributions, namely one from the supersymmetry transformation parameter λ, defined through the generalized central charge constraint (4.11), and the other one from the free coefficient μ of the separately invariant mass lagrangian (4.12). However, as we have seen, imposing invariance under parity implies that the mass is completely determined by the parameter λ. This means that, at the quantum level, where the symmetries are expressed by Ward identities (see, e.g., [10]), the total mass is then defined as a parameter of the supersymmetry Ward identities — to the contrary of the usual case of a mass introduced as a separate invariant term of the action, such as the term $\mathcal{L}_\mu$ above. It follows that, in such a situation, the mass *is not renormalized.*

However, for the latter result to hold, we must be certain that, after having set $\mu = 0$ in the action, a counterterm of the form (4.12) will not appear as a radiative correction. This can be guarantied by the presence of a "protecting" symmetry forbidding such a counterterm. From the preceding discussion on parity, we can conclude that parity itself provides a protecting symmetry. Parity invariance will thus assure the absence of any independent mass counterterm.

[5]The 4-component Dirac spinor $\Psi = (\chi_\alpha,\, \bar\psi^{\dot\alpha})$ then transforms as $\Psi \to \gamma^0\Psi$. In terms of Ψ, the fermion mass term in(4.14) is a superposition of the scalar $\bar\Psi\Psi$ and of the pseudoscalar $\bar\Psi\gamma^5\Psi$.

Remarks.

- We observe that the opposite situation, where the mass is entirely put by hand, namely the case $\lambda = 0$, $\mu \neq 0$, yields a lagrangian equivalent to the one discussed above and corresponding to $\lambda \neq 0$, $\mu = 0$. Indeed a field redefinition of the type (3.10) with $u = \pi$ just corresponds to the interchange of the mass lagrangians $\mathcal{L}_\lambda$ (4.10) and $\mathcal{L}_\mu$ (4.12). Thus, even if the mass is put by hand, it is possible to reformulate the theory in such a way that the mass is induced by the supersymmetry transformation laws, and thus would stay unrenormalized in presence of interactions.

- Of course, our "nonrenormalization theorem" is trivial in the context of the present free theory. It will however become relevant in the case of a coupling with gauge fields [8].

6. Conclusions

We have shown that the constraint on the central charges of the $N = 2$ hypermultiplet, necessary in order to keep the number of its components finite, may be generalized introducing a dimensionful complex parameter λ and a dimensionless real parameter v (eq. (3.6)). These parameters modify the supersymmetry transformation rules in a nontrivial way, but preserving the superalgebra (2.3). However, we were able to construct an invariant action only with these parameters restricted to $v = 0$ and λ purely imaginary. The latter parameter eventually contributes to the mass. A nonrenormalization theorem for the mass then follows if there is no other, independent, contribution to it. This may be asssured thanks to a protecting symmetry, which turns out to be parity. Finally, we have noted that, up to field redefinitions, the theory with mass put by hand and the theory with mass generated by supersymmetry transformation rules are equivalent.

Comparison with the results of [6, 7] is straightforward at the level of the transformation laws. Performing the auxiliary field redefinition $F \to F' = F + \frac{1}{2}\phi$ in (3.11), yields in the case of imaginary λ the transformation laws corresponding to the equation (9) of ref. [6], whith the constant field strength — denoted there by W_0 — being proportional to λ.

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JHEP03(2002)006

A. Notations and conventions

Space-time is minkovskian, 4-vector components are labelled by latin letters $a, b, \ldots = 0, 1, 2, 3$, the metric is choosen as

$$\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1) \,. \tag{A.1}$$

Weyl spinors are complex 2-component spinors ψ_α, $\alpha = 1, 2$, in the $(1/2, 0)$ representation of the Lorentz group, or $\psi_{\dot\alpha}$, $\dot\alpha = 1, 2$, in the $(0, 1/2)$ representation. The $N = 2$ internal symmetry group is "isospin" SU(2), isospinors being denoted by X^i, $i = 1, 2$.

Isospin indices i are raised and lowered by the antisymmetric tensors ε^{ij} and ε_{ij}:

$$X^i = \varepsilon^{ij} X_j \,, \qquad X_i = \varepsilon_{ij} X^j \,, \tag{A.2}$$

with:

$$\varepsilon^{ij} = -\varepsilon^{ji} \,, \qquad \varepsilon^{12} = 1 \,, \qquad \varepsilon_{ij}\varepsilon^{jk} = \delta_i^k \,, \qquad \varepsilon^{ij}\varepsilon_{kl} = \delta_l^i \delta_k^j - \delta_k^i \delta_l^j \,. \tag{A.3}$$

The same holds for the Lorentz spin indices, with the tensors $\varepsilon^{\alpha\beta}$ and $\varepsilon^{\dot\alpha\dot\beta}$ obeying to the same rules (A.3).

Multiplication of spinors and isospinors is done, if not otherwise stated, according to the convention

$$\psi\chi = \psi^\alpha \chi_\alpha \,, \qquad \bar\psi\bar\chi = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} \,, \qquad UV = U^i V_i \,. \tag{A.4}$$

Our conventions for the complex conjugation, denoted by *, are as follows:

$$(X_\alpha^i)^* = \bar X_{i\dot\alpha} \,, \qquad (\bar X_{i\dot\alpha})^* = X_\alpha^i \,. \tag{A.5}$$

The matrices σ^a and $\bar\sigma^a$ are defined by

$$\begin{aligned}
&\bar\sigma^{a\,\dot\alpha\alpha} = \varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\sigma^a_{\beta\dot\beta} \,, \\
&\sigma^0 = \mathbb{1} \,, \qquad \sigma^i \; (i = 1, 2, 3) \; = \text{Pauli matrices} \,, \\
&\bar\sigma^0 = \mathbb{1} \,, \qquad \bar\sigma^i \; (i = 1, 2, 3) \; = -\text{Pauli matrices} \,,
\end{aligned} \tag{A.6}$$

and obey the properties

$$\sigma^a\bar\sigma^b + \sigma^b\bar\sigma^a = -2\eta^{ab} \,, \qquad \sigma^a_{\alpha\dot\alpha}\bar\sigma_a^{\dot\beta\beta} = -2\delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta} \,. \tag{A.7}$$

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JHEP03(2002)006